



Ingegneria delle Telecomunicazioni

Satellite Communications

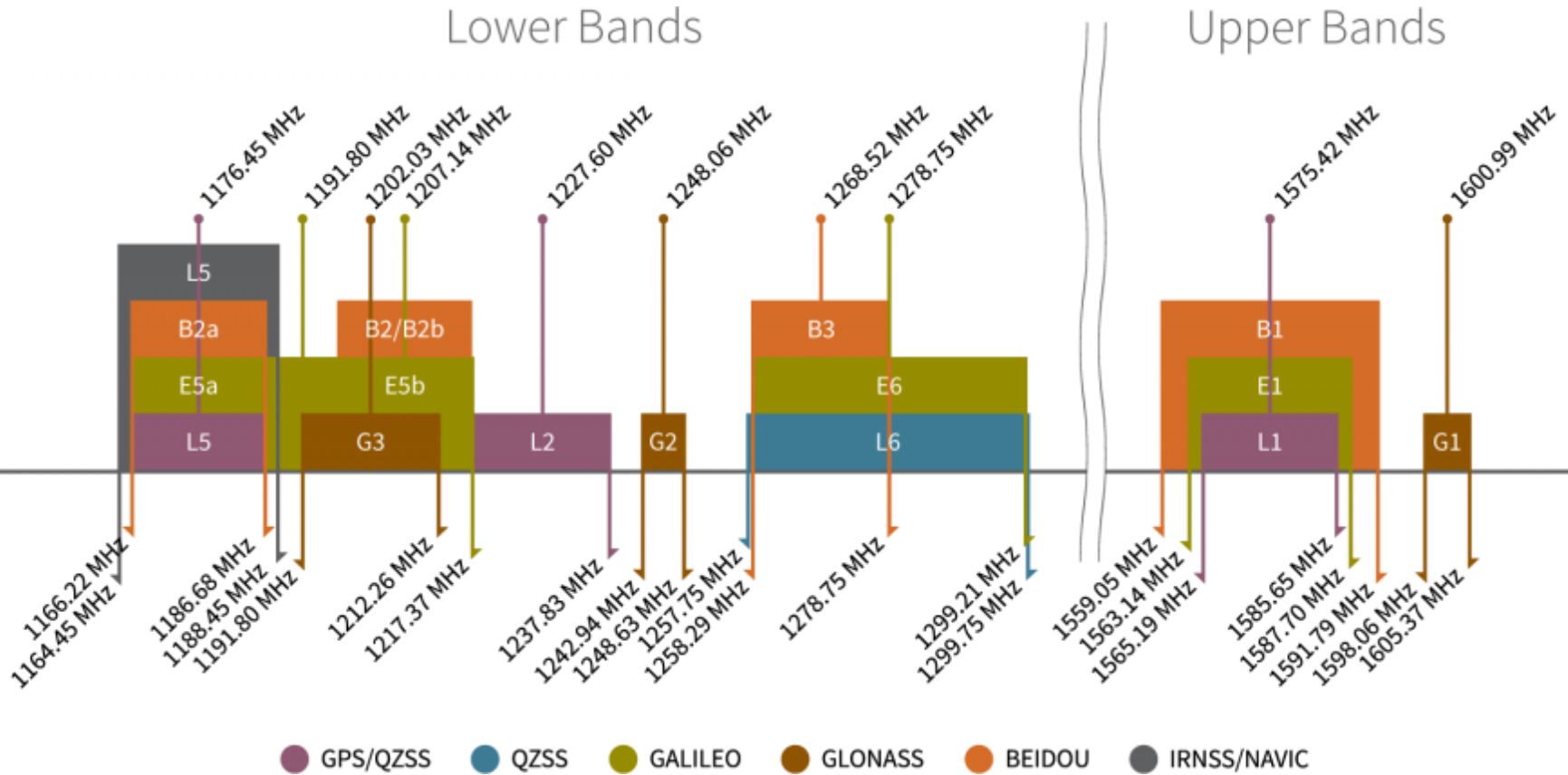
20. From Outer Space to Earth – GNSS Bands & Signals

Marco Luise

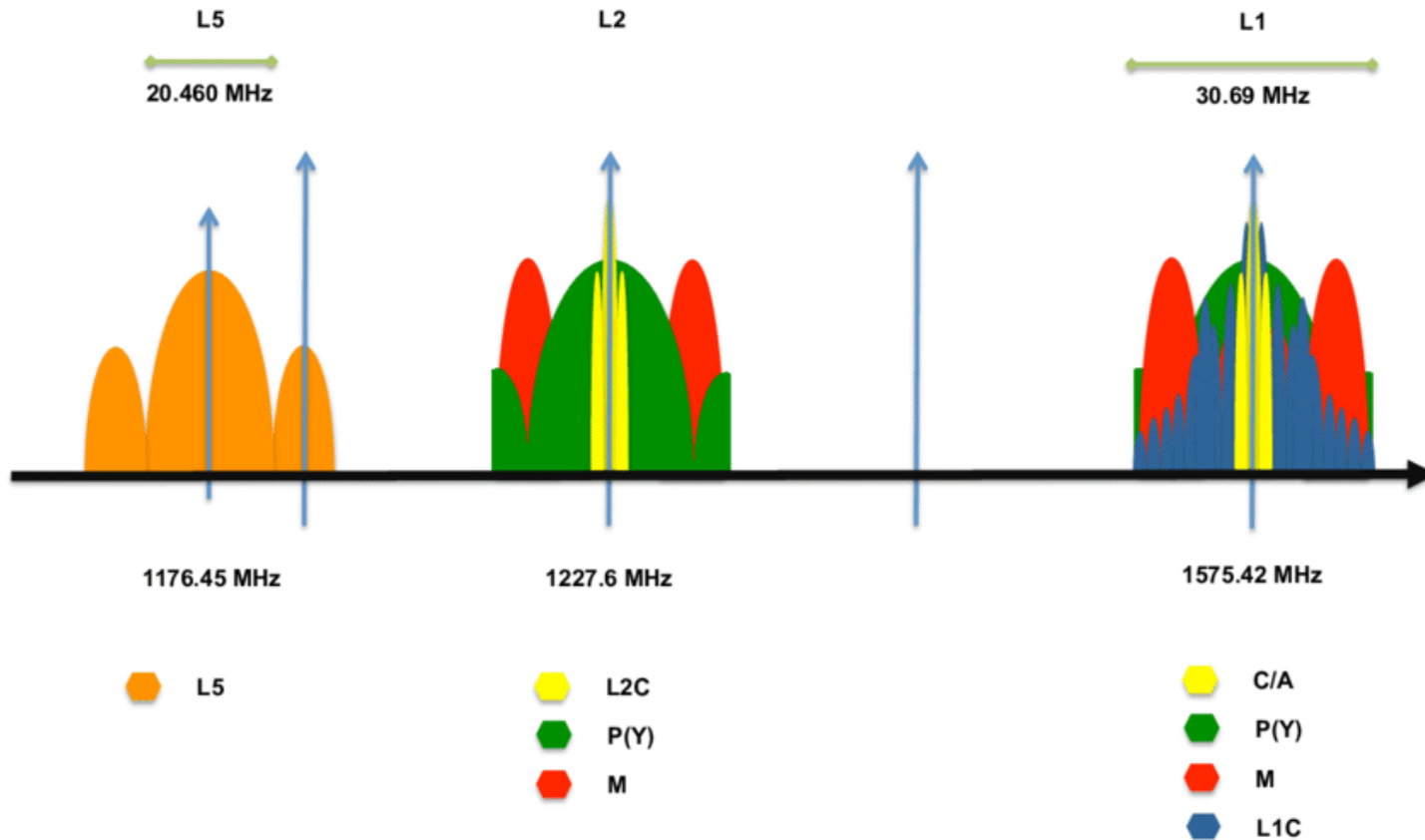
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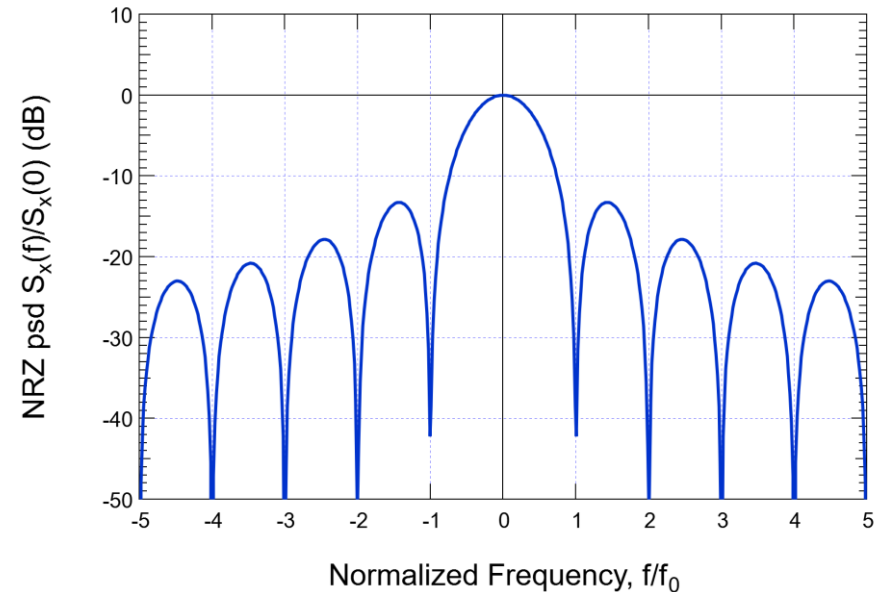
Satellite Navigation Frequency Bands & Systems



GPS Signals: L1 C/A and L2C



- Carrier Frequency: $f_c=1575.42$ MHz = $1540 f_0$, $f_0=1.023$ MHz (L1)
- # Components: 1
- Bit Rate: 50 bps
- Data Protection Coding: None
- Chip Rate: $R_c=f_0$
- Modulation/Spreading: DS/SS BPSK with NRZ chip pulse $p(t)$
- Type/Length of Ranging Code: Satellite-specific Gold Code $L=1023$

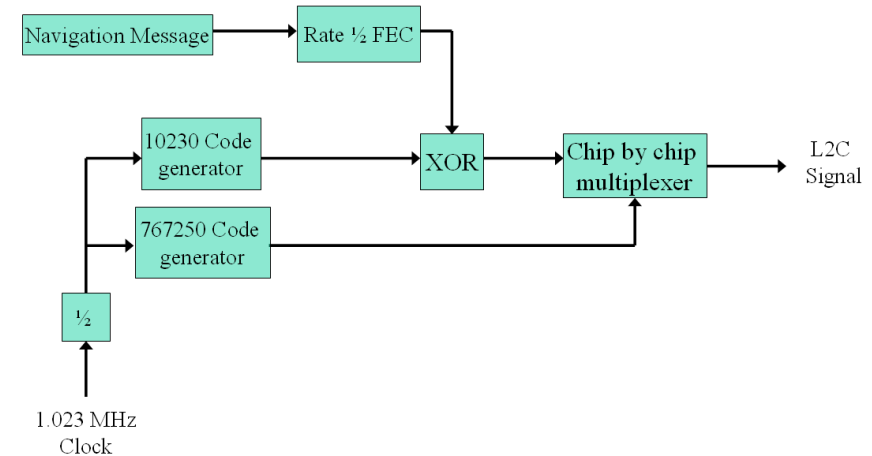


$$x_{C/A}(t) = \sum_n c_{C/A}[n] d_{C/A}[n // 20460] p(t - nT_c) + j0$$

n is the chip index; $k=n//20460$ is the bit index, where $n//20460$ means «the result of the integer division $n/20460$ »

(Modernized) GPS L2C: Pilot Signal & Data Encoding

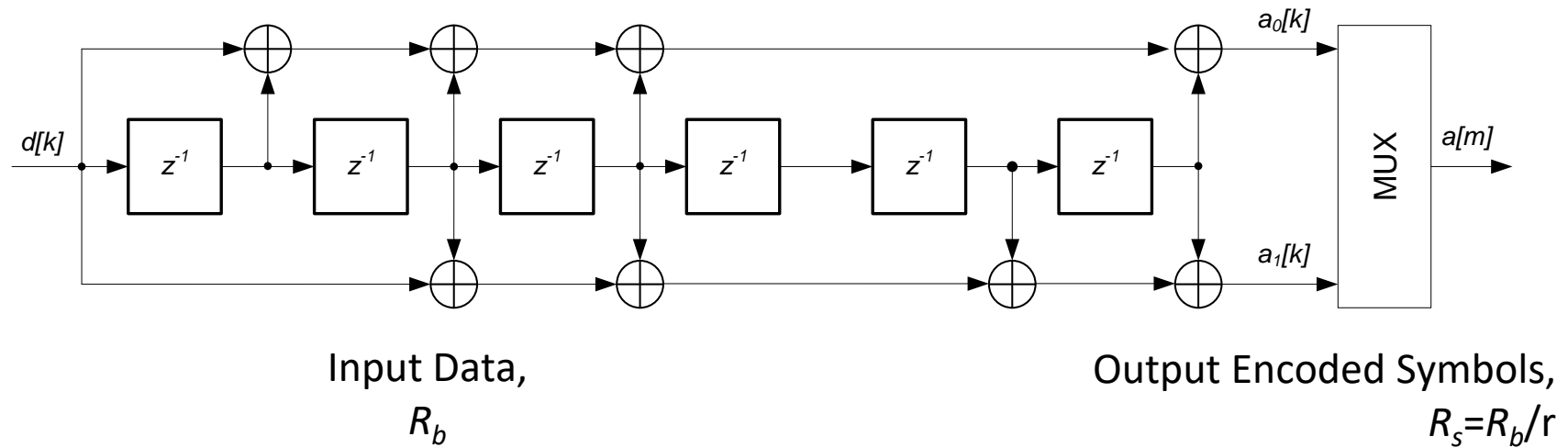
- Carrier Frequency:
 $f_c = 1227.60 \text{ MHz} = 1200 f_0$
- # Components: 2 (Data and Pilot)
- Data Bit Rate: 25 bps
- Data Coding: $K=7$, $r=1/2$ convolutional encoding, symbol rate $R_s=50$ baud
- Chip Rate: $R_c=f_0$ time-multiplex of the two components at $f_0/2$
- Modulation/Spreading: DS/SS BPSK with NRZ chip pulse $p(t)$ – same spectrum as C/A
- Pilot Code (no data): Civil-Long Code with $L=767250$ (1.5 s) extracted from a longer ML sequence, Data channel: Civil-Short $L=10230$ extracted from the same ML



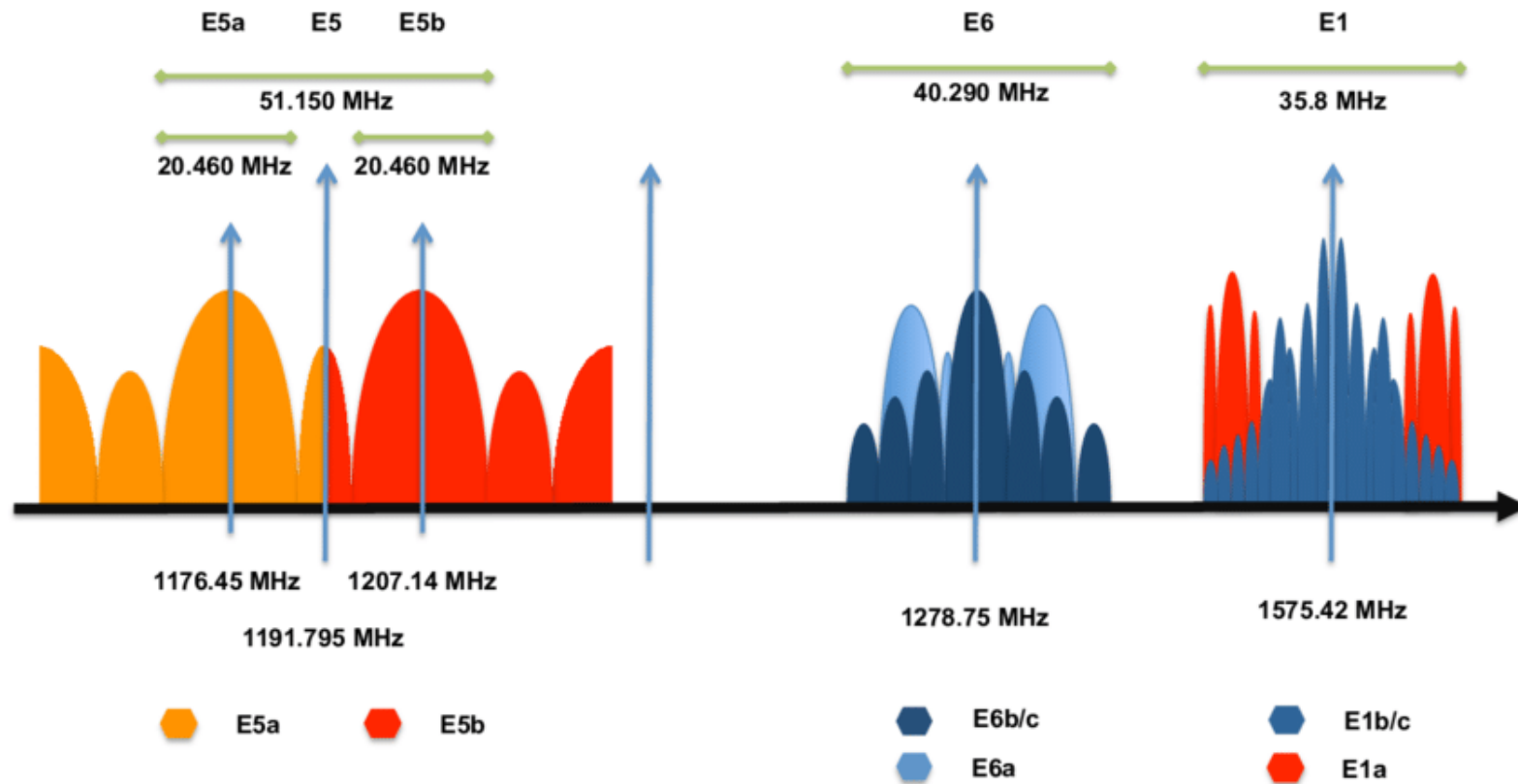
$$x_{L2c}(t) = \sum_m c_{CS}[m] a_{L2c}[m / 5115] p(t - 2mT_c) + \sum_m c_{CL}[m] p(t - (2m+1)T_c) + j0$$

$a_{L2c}[m]$ encoded symbol

Convolutional Encoding

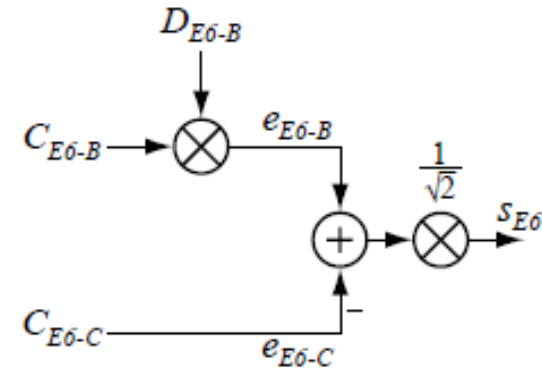


GALILEO Signals: E6 B/C



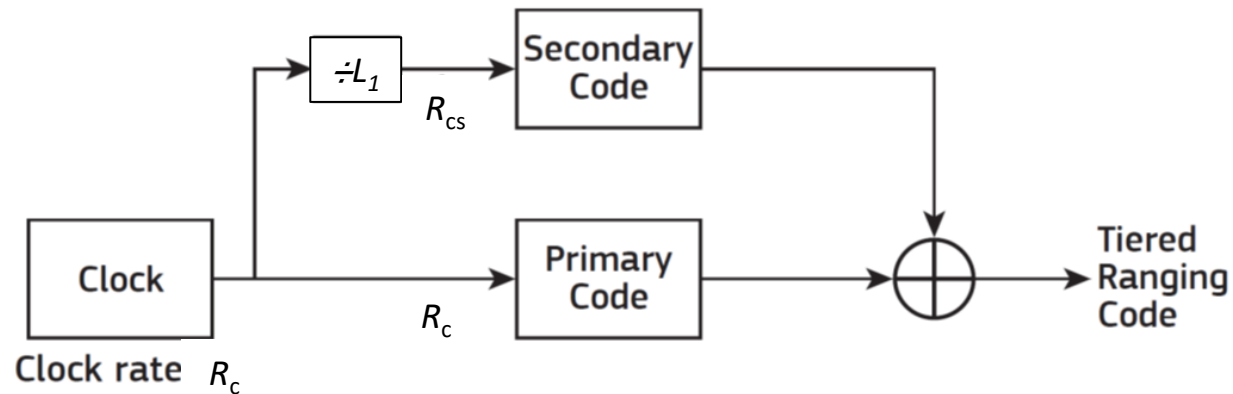
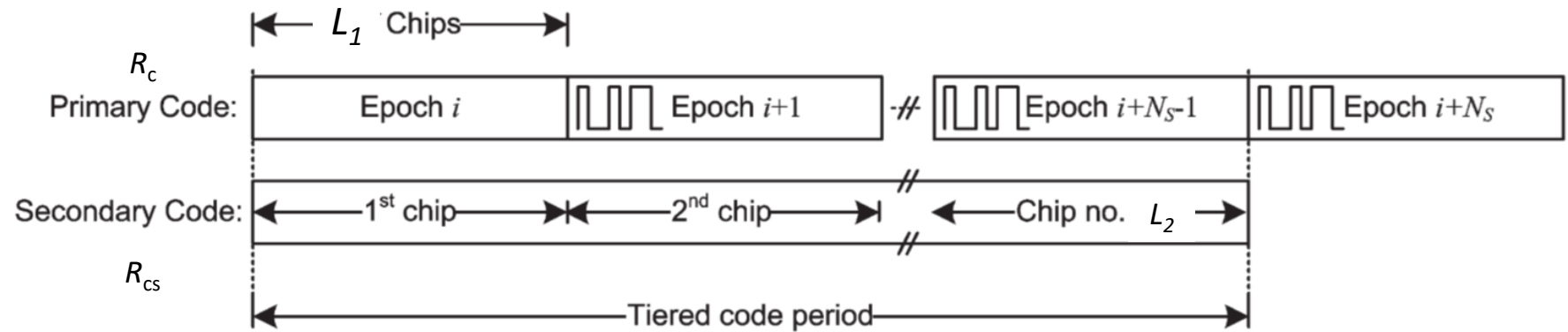
GALILEO E6 B/C: primary/secondary codes

- Carrier Frequency: $f_c = 1278.75 \text{ MHz} = 1250 f_0$
- # Components: 2 (B and C channels)
 - B channel Bit Rate: $R_b = 500 \text{ bps}$
 - B channel Data Coding: Convolutional, $r = 1/2$, symbol rate $R_s = 1,000 \text{ baud}$
 - C channel: no data (pilot channel, pure code)
- Chip Rate: $R_c = 5f_0 = 5.115 \text{ Mchip/s}$
- Modulation/Spreading: DS/SS BPSK with NRZ chip pulse $p(t)$ – same spectrum as GPS L1 on a wider $5f_0$ bandwidth.
- Type/Length of Ranging Code:
 - B channel: memory code $L = 5115$
 - C channel: memory code $L = 5115$ XOR secondary memory code with a chip time equal to the primary code repetition period (same as data symbol rate)



$$x_{E6}(t) = \frac{1}{\sqrt{2}} \sum_n c_{E6B}[n] a_{E6}[n/5115] p(t - nT_c) + \frac{1}{\sqrt{2}} \sum_n (c_{E6C,p}[n] c_{E6C,s}[n/5115]) p(t - nT_c) + j0$$

Primary and secondary Codes (Tiered Code)



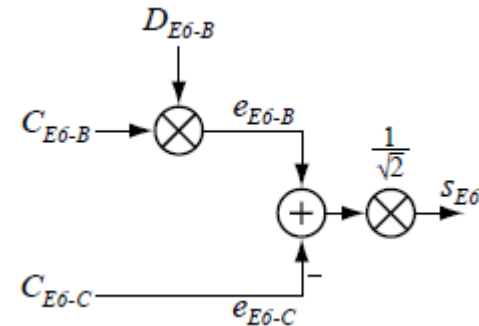
Overall LONG code (good correlation properties), yet simpler to acquire (primary)

Code Lengths

Signal Component	Tiered Code Period (ms)	Code Length (chips)	
		Primary	Secondary
E5a-I	20	10230 (1 ms)	20
E5a-Q	100	10230 (1 ms)	100
E5b-I	4	10230 (1 ms)	4
E5b-Q	100	10230 (1 ms)	100
E1-B	4	4092 (4 ms)	N/A
E1-C	100	4092 (4 ms)	25

Galileo E6 – from Signal-In-Space (SIS) ICD

Signal composition:

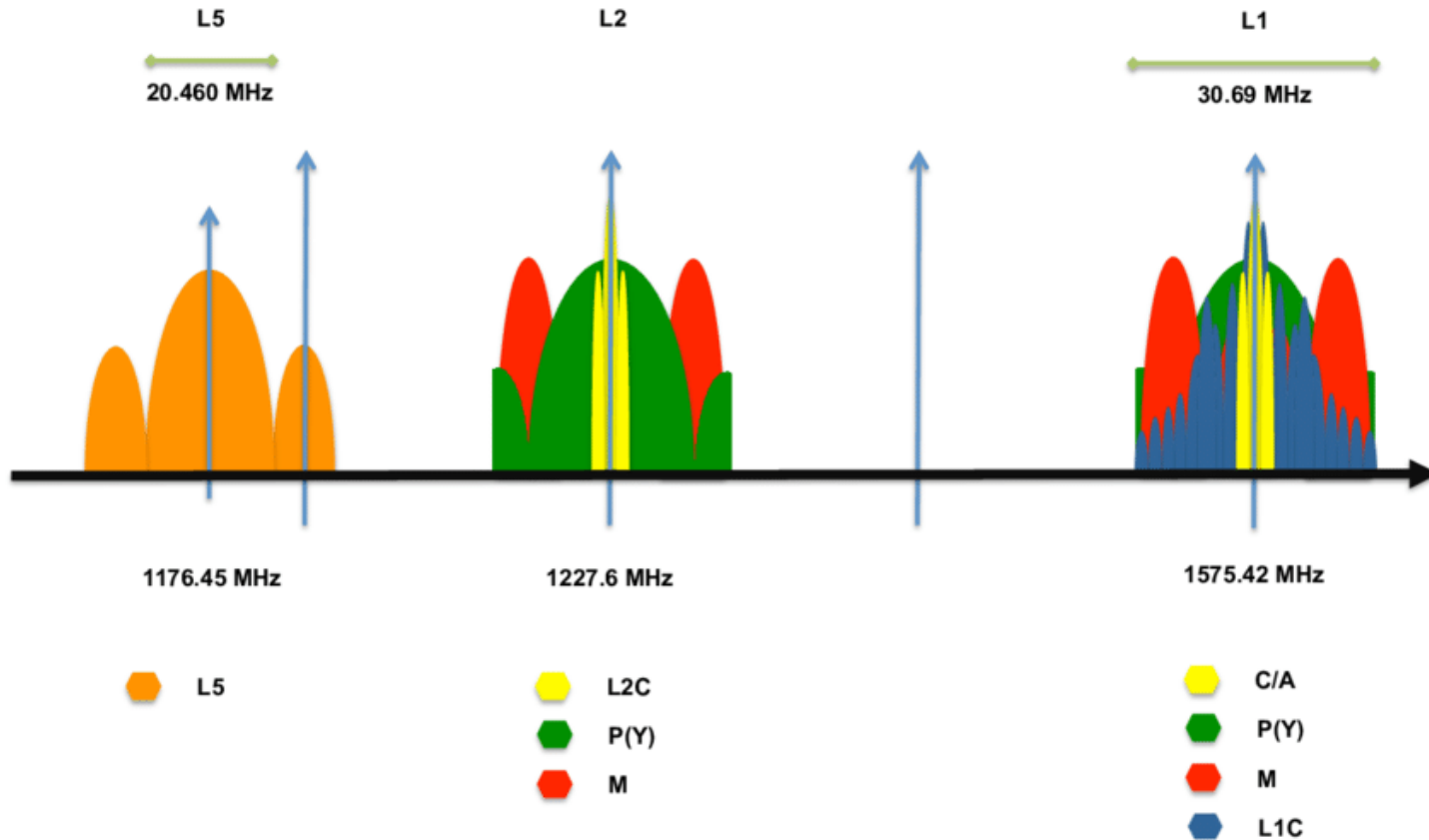


$$e_{E6-B}(t) = \sum_{i=-\infty}^{+\infty} \left[c_{E6-B,|i|_{L_{E6-B}}} d_{E6-B,[i]_{DC_{E6-B}}} \text{rect}_{T_{C,E6-B}}(t - iT_{C,E6-B}) \right]$$

$$e_{E6-C}(t) = \sum_{i=-\infty}^{+\infty} \left[c_{E6-C,|i|_{L_{E6-C}}} \text{rect}_{T_{C,E6-C}}(t - iT_{C,E6-C}) \right]$$

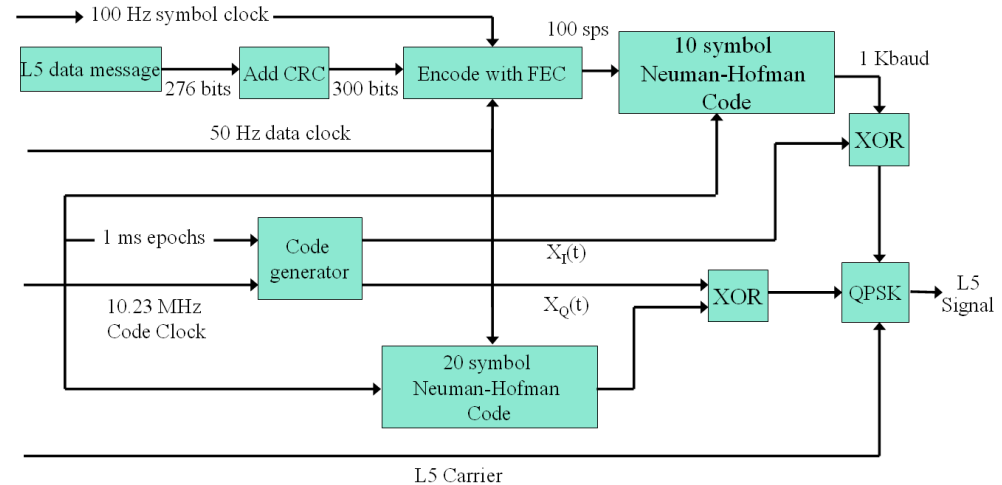
Component (Parameter Y)	Ranging Code Chip-Rate $R_{C,E6-Y}$ (MChip/s)	Symbol-Rate $R_{D,E6-Y}$ (symbols/s)
B	5.115	1000
C	5.115	No data ('pilot component')

(Modernized) GPS L5



(Modernized) GPS L5: pilot, coding, primary/secondary, wideband

- Carrier Frequency: $f_c=1176.45$ MHz = $1150 f_0$
- # Components: 2
 - I : Bit Rate/Coding: $R_b=50$ bps, $r=1/2$ convolutional, symbol rate $R_s=100$ baud
 - Q : pilot
- Chip Rate: $R_c=10f_0=10.23$ Mchip/s
- Modulation/Spreading: DS/SS QPSK with NRZ chip pulse $p(t)$ – same spectrum as GPS L1 on a wider $10f_0$ bandwidth.
- Type/Length of Ranging Code:
 - I : Primary M-sequence, $L_1=10230$ (1 ms), secondary $L_2=10$ -symbol short code @ 1 kbaud
 - Q : same primary as above with time shift, secondary length 20 symbols.



$$x_{L5}(t) = \frac{1}{\sqrt{2}} \sum_n (c_{I5}[n]c_{OI5}[n//10230]) a_{L2c}[n//102300] p(t - nT_c) + j \frac{1}{\sqrt{2}} \sum_n c_{Q5}[n]c_{OQ5}[n//10230] p(t - nT_c)$$

The MCRB for pseudorange accuracy 1/2

$$\sigma_{\tau} [\text{m}] \geq cT_c \times \sqrt{\frac{B_L}{2C/N_0} \left(\frac{1}{2\pi\beta} \right)}$$

$B_L = 1/(2T_0)$ loop bandwidth (we'll see later on what it means) [Hz]

C/N_0 signal-to-noise-ratio per unit bandwidth [dB · Hz]

cT_c equivalent chip length [m]

$S_s(f)$ GNSS signal psd

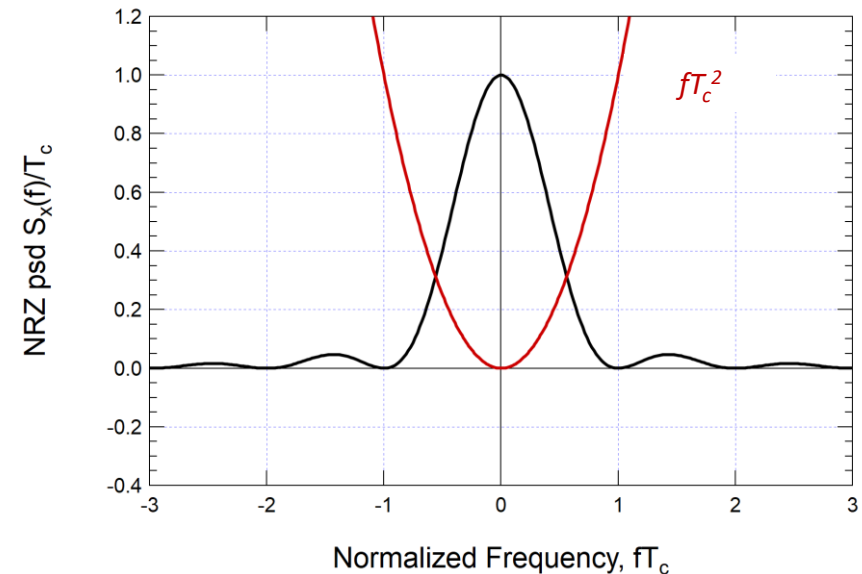
$$\beta^2 = \frac{T_c^2 \int_{-B_{RF}/2}^{+B_{RF}/2} f^2 S_s(f) df}{\int_{-B_{RF}/2}^{+B_{RF}/2} S_s(f) df}$$

Normalized Squared Gabor Bandwidth in
the receiver (radio) bandwidth B_{RF}

The MCRB for pseudorange accuracy 2/2

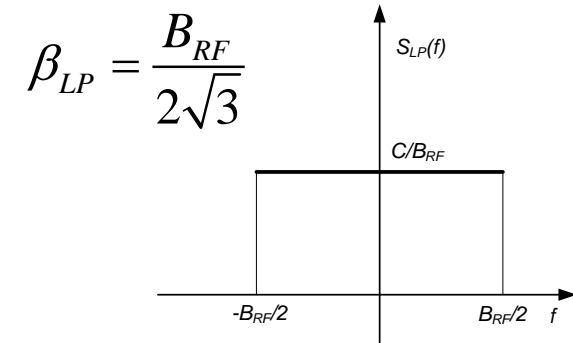
$$\beta^2 = \frac{T_c^2 \int_{-B_{RF}/2}^{+B_{RF}/2} f^2 S_s(f) df}{\int_{-B_{RF}/2}^{+B_{RF}/2} S_s(f) df}$$

- The larger the Gabor bandwidth, the more accurate the pseudorange estimate
- The Gabor bandwidth is roughly proportional to the signal bandwidth **BUT**
- A conventional NRZ spectrum with most of its energy at the carrier frequency is not optimal



1. Low-Pass Spectrum

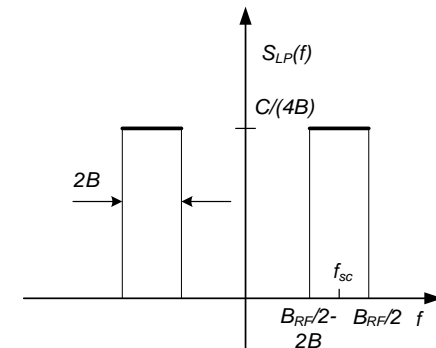
$$\beta_{LP}^2 = \frac{\int_{-B_{RF}/2}^{B_{RF}/2} f^2 \frac{C}{B_{RF}} df}{C} = \frac{f^3}{3B_{RF}} \Big|_{-B_{RF}/2}^{B_{RF}/2} = \frac{B_{RF}^2}{12}$$



2. Band-Pass Spectrum within the baseband (spectrum with subcarriers)

$$\beta_{SC}^2 \triangleq \frac{2 \int_{B_{RF}/2-2B}^{B_{RF}/2} f^2 \frac{C}{4B} df}{C} =$$

$$= \beta_{LP}^2 \left(3 - 12 \frac{B}{B_{RF}} + 16 \left(\frac{B}{B_{RF}} \right)^2 \right)$$

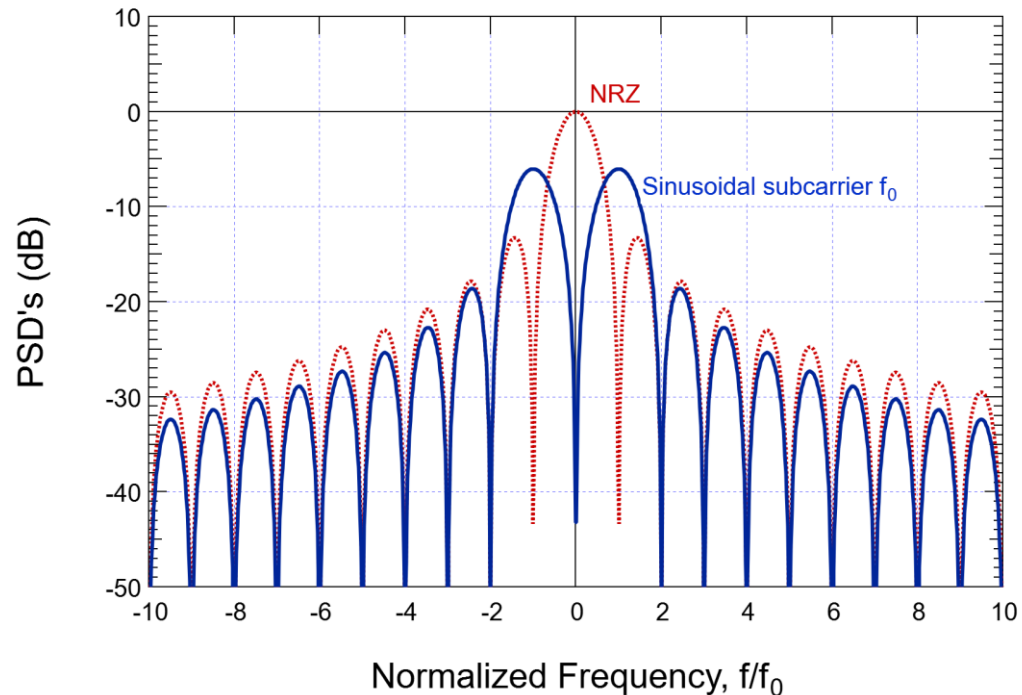


Creating a Subcarrier (Offset) Signal

Starting from GPS C/A and adding subcarriers @ $\pm f_0$...

$$x(t) = \frac{1}{2j} \left(s(t)e^{j2\pi f_0 t} - s(t)e^{-j2\pi f_0 t} \right) = s(t) \sin(2\pi f_0 t) = \sin(2\pi f_0 t) \sum_n c[n] d[n/M] p(t - nT_c)$$

Better Gabor bandwidth
And little interference
with NRZ

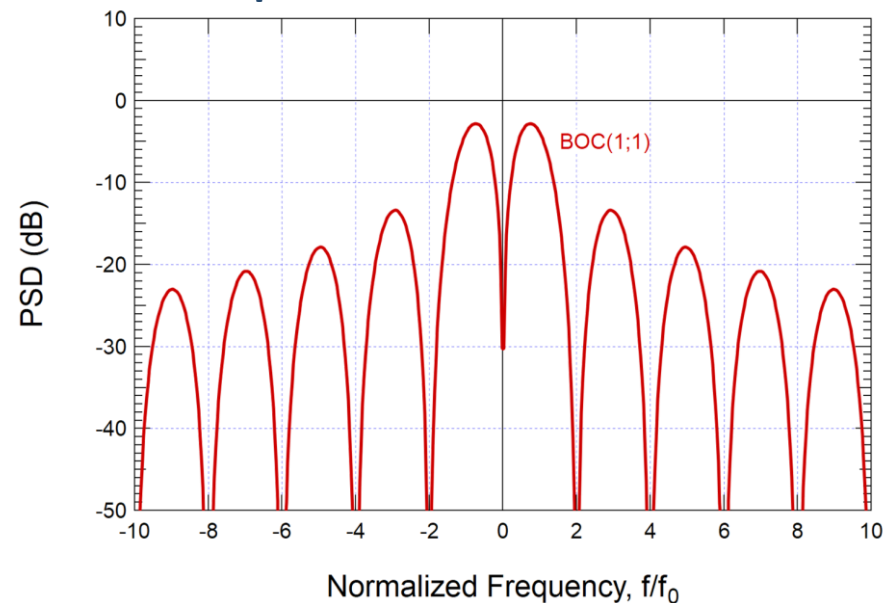


Creating a Binar Offset Carrier (BOC) Signal

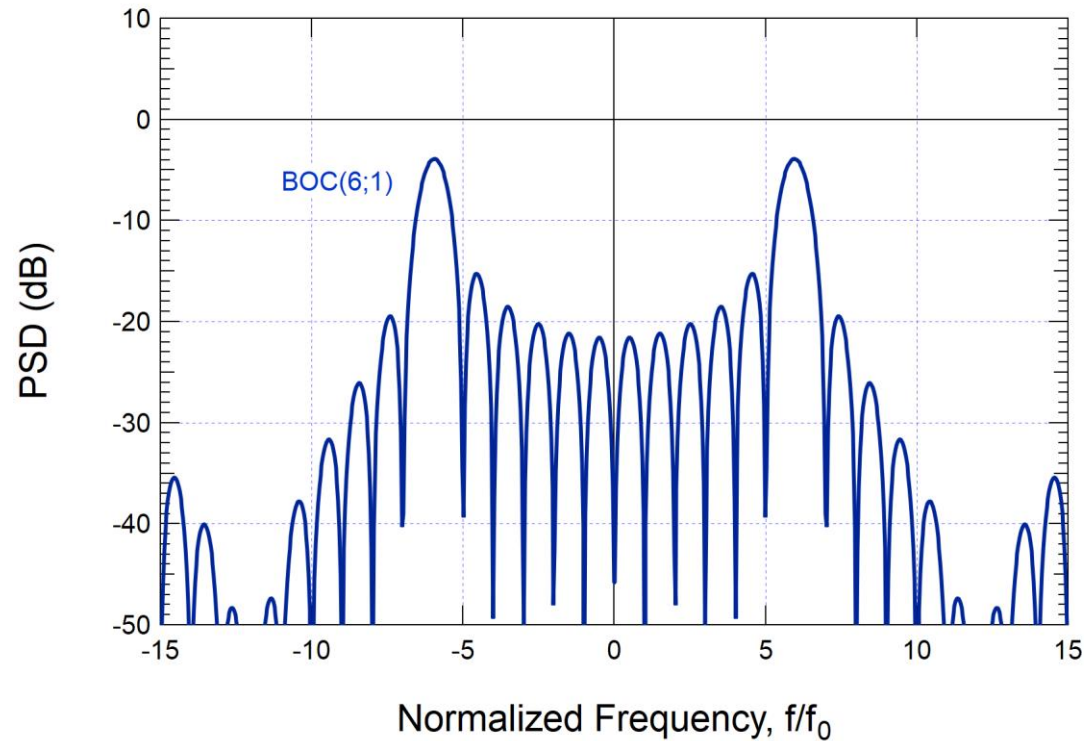
The Offset signal is no longer constant-amplitude: was ± 1 only, now any value between -1 and +1. So instead of the sinusoidal subcarrier we use a **square-wave**, **binary subcarrier**

$$x(t) = s(t) \operatorname{sgn} \left[\sin(2\pi f_0 t) \right]$$

The spectrum is different than before, but it is not so different, still *offset*, AND the signal is constant-envelope !



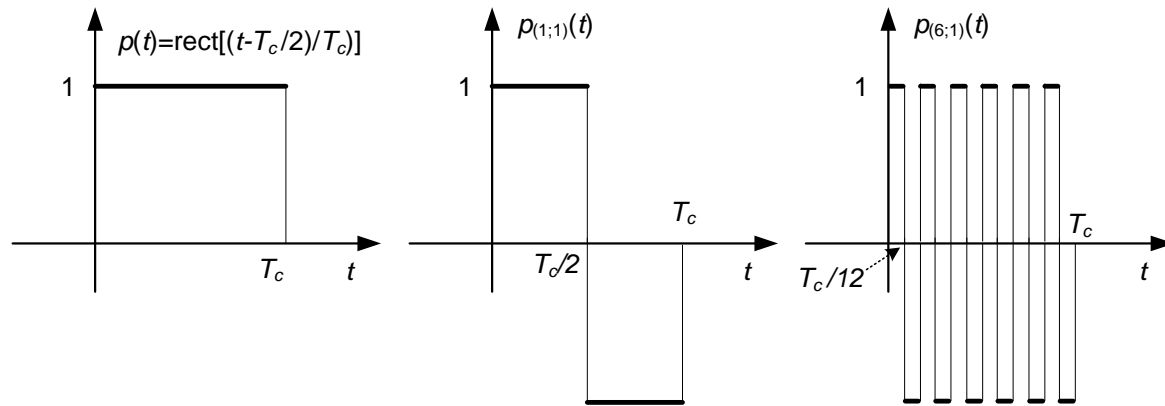
BOC(n ; m) means: chip rate mf_0 and subcarrier frequency nf_0



They are massively used in any GNSS

BOC Spectrum (n/m INTEGER)

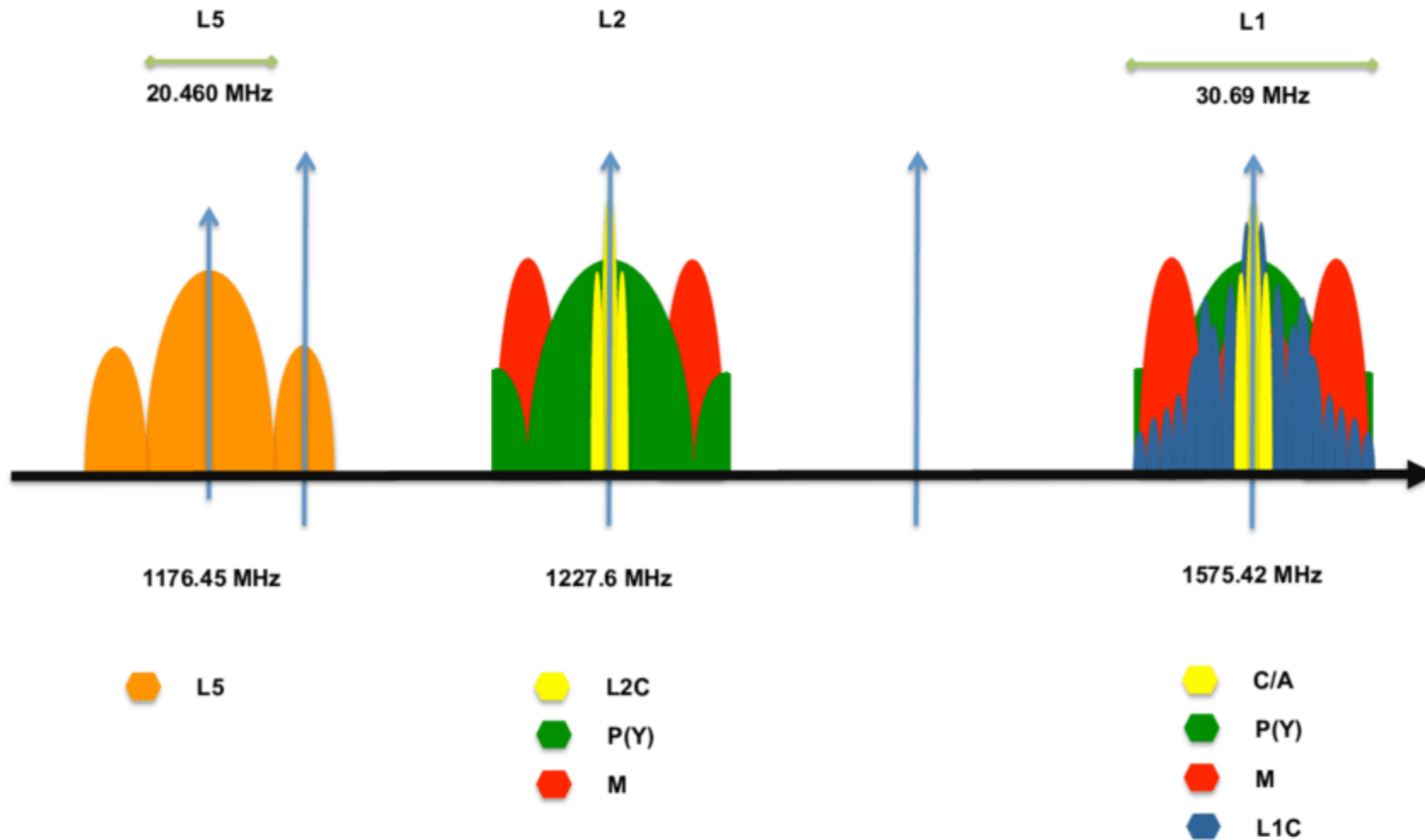
Easy computation of the BOC(n;m) spectrum: “embedding” the subcarrier into the chip pulse $p(t)$



$$S_{(1;1)}(f) = \frac{1}{T_c} |P_{(1;1)}(f)|^2 = f_0 \left| \frac{1}{2f_0} \operatorname{sinc}\left(\frac{f}{2f_0}\right) - \frac{1}{2f_0} \operatorname{sinc}\left(\frac{f}{2f_0}\right) e^{-j\pi f T_c} \right|^2 = \frac{1}{f_0} \operatorname{sinc}^2\left(\frac{f}{2f_0}\right) \sin^2\left(\frac{\pi f}{2f_0}\right)$$

$$S_{(6;1)}(f) = \frac{1}{T_c} |P_{(6;1)}(f)|^2 = \frac{1}{T_c} \left| \sum_{\ell=0}^5 \frac{1}{6} P_{(1;1)}\left(\frac{f}{6}\right) e^{-j2\pi \ell f \frac{T_c}{6}} \right|^2 = \frac{1}{36f_0} \operatorname{sinc}^2\left(\frac{f}{12f_0}\right) \sin^2\left(\frac{\pi f}{12f_0}\right) \left[\frac{\sin(\pi f / f_0)}{\sin\left(\frac{\pi f}{6f_0}\right)} \right]^2$$

GPS Signals: L1C



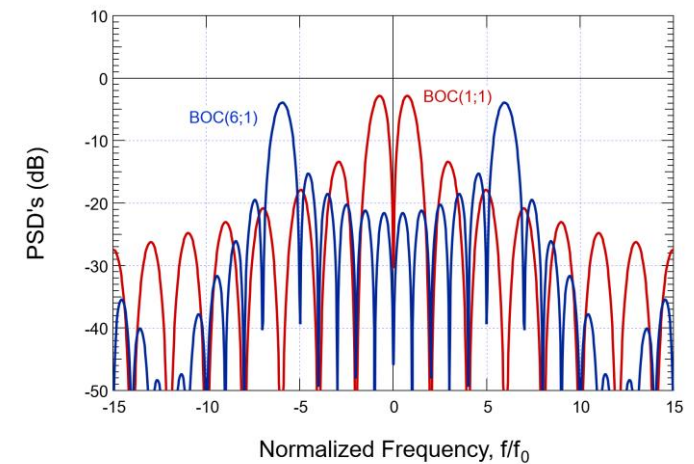
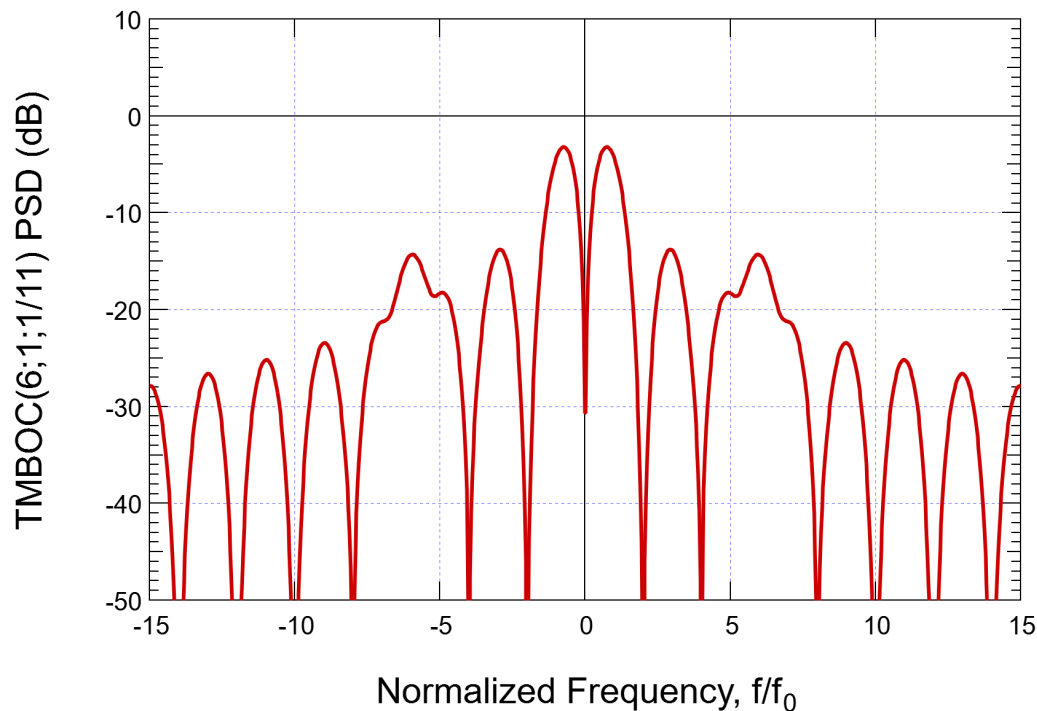
- Carrier Frequency: L1 $f_c=1575.42$ MHz= $1540 f_0$
- # Components: 2 (Data and Pilot channels)
 - Data channel Bit Rate: $R_b=50$ bps with conv., $r=1/2$ encoding, symbol rate $R_s=100$ baud
 - Pilot channel: no data
- Chip Rate: $R_c=f_0$
- Modulation/Spreading: for data, BOC(1,1); for pilot, time-multiplex of BOC(1;1) and BOC(6;1) (TMBOC), where for 29/33 of time the signal is (1;1), switching to (6;1) for 4/33 of the time to provide an occasional wider-bandwidth component
- Type/Length of Ranging Code:
 - Data channel: *extended Legendre sequence* $L_1=10230$ (10 ms, symbol period)
 - *Pilot channel: primary (different) extended Legendre sequence* $L_1=1023$, secondary ML code $L_2=1800$ with the same clock as encoded symbols (10 ms)

$$x_{E6}(t) = \frac{1}{\sqrt{2}} \sum_n c_{E6B}[n] a_{E6}[n/5115] p(t - nT_c) + \frac{1}{\sqrt{2}} \sum_n (c_{E6C,p}[n] c_{E6C,s}[n/5115]) p(t - nT_c)$$

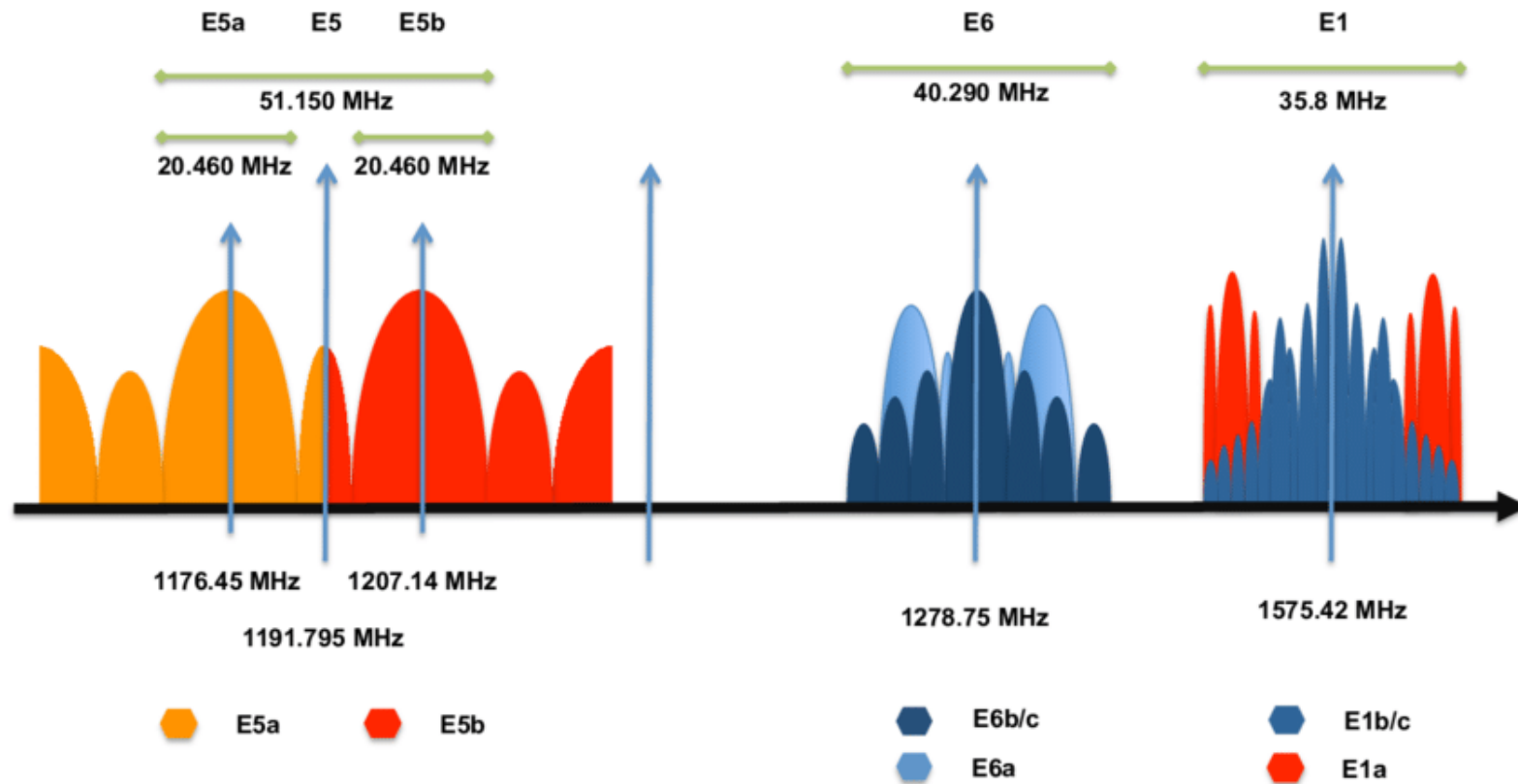
Spectrum of GPS L1C

The pilot is 5 dB stronger (*power factor 3*) than the data component:

$$S_{L1C}(f) = \frac{3}{4} S_{L1CP}(f) + \frac{1}{4} S_{L1CD}(f) = \frac{3}{4} \left(\frac{29}{33} S_{(1;1)}(f) + \frac{4}{33} S_{(6;1)}(f) \right) + \frac{1}{4} S_{(1;1)}(f) = \frac{10}{11} S_{(1;1)}(f) + \frac{1}{11} S_{(6;1)}(f)$$



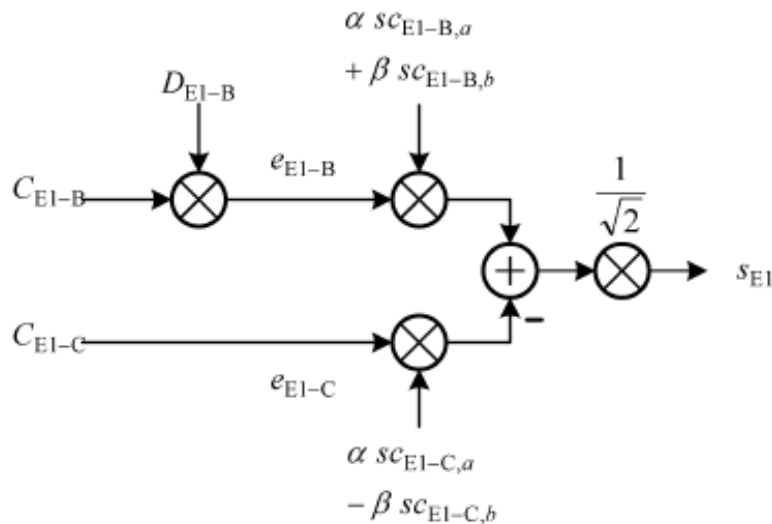
GALILEO Signals: E1 B/C



GALILEO E1 B/C: MBOC

- Carrier Frequency: L1 $f_c=1575.42$ MHz= $1540 f_0$
- # Components: 2 (Data and Pilot)
 - B (Data channel) Bit Rate: $R_b=125$ bps with conv., $r=1/2$ encoding, symbol rate $R_s=250$ baud
 - C Pilot channel: no data
- Chip Rate: R_c
- Modulation/Spreading: Composite BOC (aka Multiplexed BOC, MBOC); each channel has a different subcarrier waveform obtained as a (different) combination of BOC(1;1) and BOC(6;1) – **see next slide and pray in advance**
- Type/Length of Ranging Code:
 - Data channel: *Memory sequence* $L_1=4096$ (4 ms)
 - *Pilot channel: primary (different) Memory sequence* $L_1=4096$, secondary memory sequence $L_2=25$ with the same clock as encoded symbols (4 ms)

Galileo E1 C/B – from SIS ICD



The E1 CBOC signal components are generated as follows:

- $eE1-B$ from the I/NAV navigation data stream $DE1-B$ and the ranging code $CE1-B$, then modulated with the sub-carriers $scE1-B,a$ and $scE1-B,b$
- $eE1-C$ (pilot component) from the ranging code $CE1-C$ including its secondary code, then modulated with the sub-carriers $scE1-C,a$ and $scE1-C,b$

Component (Parameter Y)	Sub-carrier Type	Sub-carrier Rate		Ranging Code Chip-Rate $R_{C,E1-Y}$ (Mcps)
		$R_{S,E1-Y,a}$ (MHz)	$R_{S,E1-Y,b}$ (MHz)	
B	CBOC, in-phase	1.023	6.138	1.023
C	CBOC, anti-phase	1.023	6.138	1.023

Component (Parameter Y)	Symbol Rate $R_{D,E1-Y}$ (symbols/s)
B	250
C	No data ('pilot component')

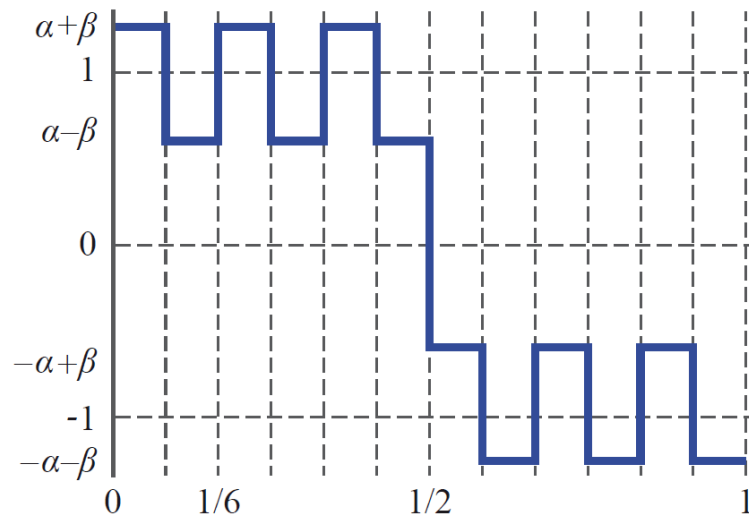
$$\alpha = \sqrt{10/11} \quad \beta = \sqrt{1/11}$$

GALILEO E1 B/C signal form

$$x_{E1}(t) = \frac{1}{\sqrt{2}} sc_B(t) \sum_n c_{E1B}[n] a_{E6}[n/4092] p(t - nT_c) + \frac{1}{\sqrt{2}} sc_C(t) \sum_n (c_{E6C,p}[n] c_{E6C,s}[n/4092]) p(t - nT_c) + j0$$

Subcarrier for B

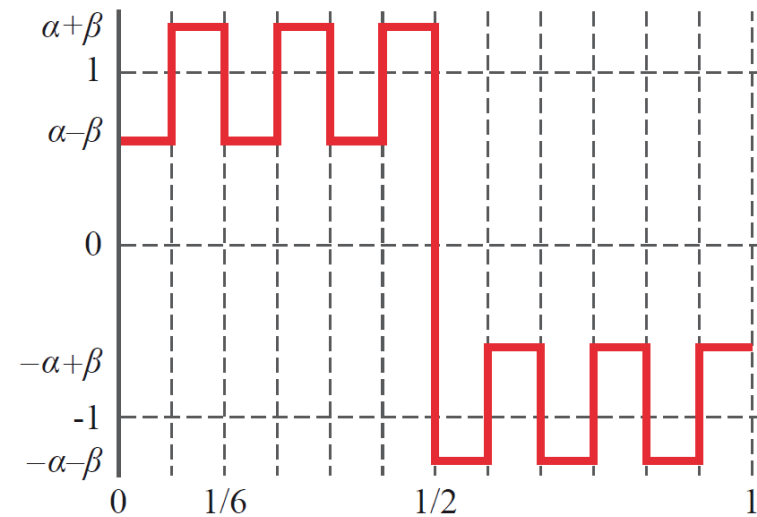
$$sc_B(t) = \sqrt{\frac{10}{11}} \operatorname{sgn}[\sin(2\pi f_0 t)] + \sqrt{\frac{1}{11}} \operatorname{sgn}[\sin(2\pi 6 f_0 t)]$$


 $t/T_{c,E1-B}$

$$\alpha = \sqrt{10/11} \quad \beta = \sqrt{1/11}$$

Subcarrier for C

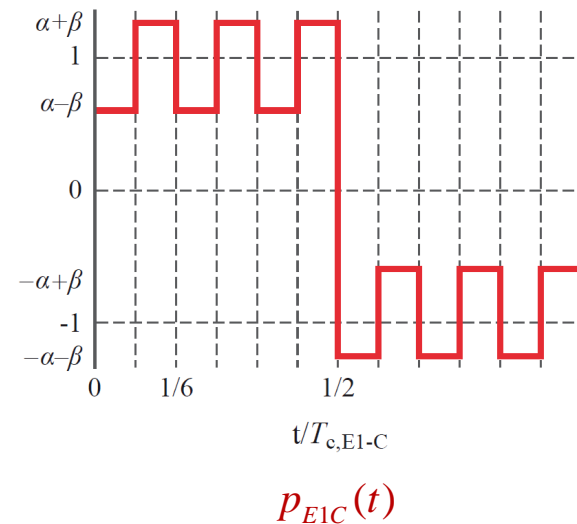
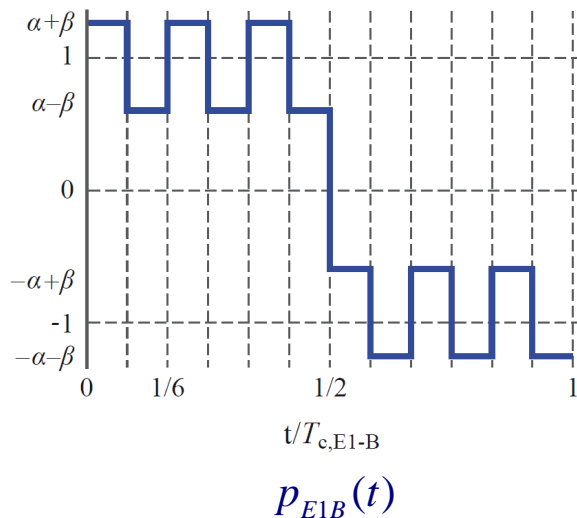
$$sc_C(t) = \sqrt{\frac{10}{11}} \operatorname{sgn}[\sin(2\pi f_0 t)] - \sqrt{\frac{1}{11}} \operatorname{sgn}[\sin(2\pi 6 f_0 t)]$$


 $t/T_{c,E1-C}$

To compute the Spectrum of E1...

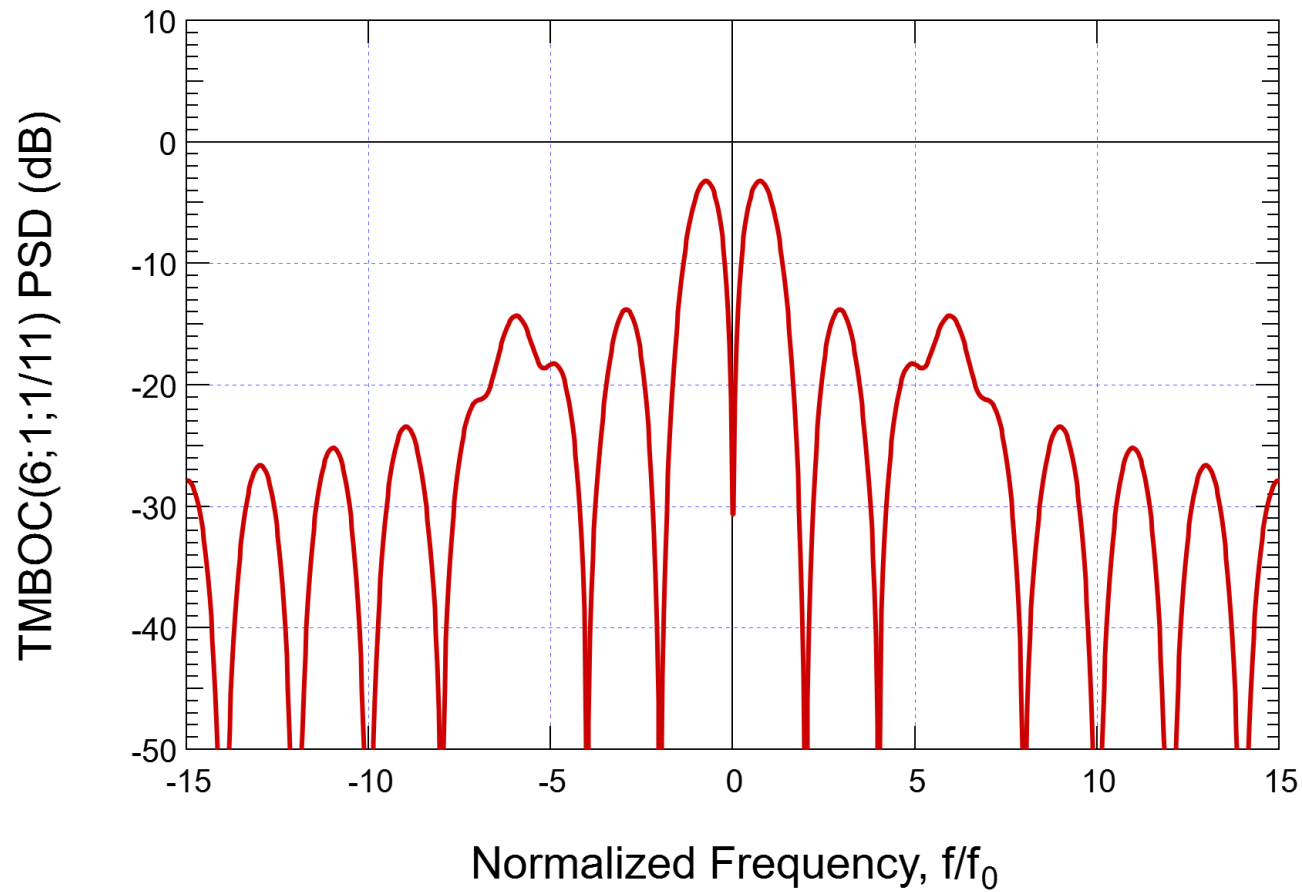
The subcarriers can be “embedded” into the chip pulses, that are now different for the two components

$$\begin{aligned}
 x_{E1}(t) &= \frac{1}{\sqrt{2}} sc_B(t) \sum_n c_{E1B}[n] a_{E6}[n/4092] p(t - nT_c) + \frac{1}{\sqrt{2}} sc_c(t) \sum_n (c_{E6C,p}[n] c_{E6C,s}[n/4092]) p(t - nT_c) + j0 \\
 &= \frac{1}{\sqrt{2}} \sum_n c_{E1B}[n] a_{E6}[n/4092] p_{E1B}(t - nT_c) + \frac{1}{\sqrt{2}} \sum_n (c_{E6C,p}[n] c_{E6C,s}[n/4092]) p_{E1C}(t - nT_c) + j0
 \end{aligned}$$



SAME SPECTRUM AS L1C

L1C & E1 Spectrum



And finally: GALILEO E5a-b

- Carrier Frequency: L1 $f_c=1,191.795$ MHz= $1165 f_0$
- # Components: 4 (2 x Data (a,b) on I and 2x Pilot (a,b) on Q)
 - Data channels Bit Rate: $R_b=25$ bps on $a-I$, 50 bps on $b-I$, conv., $r=1/2$ encoding, symbol rate $R_s=125$ baud on $a-I$, 250 bps on $b-I$
- Chip Rate: $R_c=10f_0$ (All)
- Modulation/Spreading: AltBOC (**wait and fear**) with two subcarriers to be possibly received *separately* and carrying different channels each, or to be processed *jointly* on a very wide bandwidth (50 MHz)
- Type/Length of Ranging Codes:
 - Primary codes length (all channels): $L_1=10230$ (1 ms). Generated through LFSR with different parameters (*Truncated and Combined M-sequences*)
 - Secondary codes length: $L_2=20$ for $a-I$, $L_2=4$ for $b-I$. $L_2=100$ for pilots. They are all memory codes.

We can create a two-channel *single side-band* signal (called *a*) from baseband I/Q components using a complex subcarrier for example at subcarrier frequency $15f_0$:

$$x_a(t) = \left[x_{a-I}(t) + jx_{a-Q}(t) \right] e^{-j2\pi 15f_0 t}$$

And we can also add *another* two-channel component (called *b*) at $-15f_0$:

$$x(t) = \left[x_{a-I}(t) + jx_{a-Q}(t) \right] e^{-j2\pi 15f_0 t} + \left[x_{b-I}(t) + jx_{b-Q}(t) \right] e^{+j2\pi 15f_0 t}$$

On each component, the two channels have different codes and can be CDMA-separated; if they are narrow-band enough, the two subcarriers can be FDMA-separated as well .

BUT the signal is not constant-amplitude

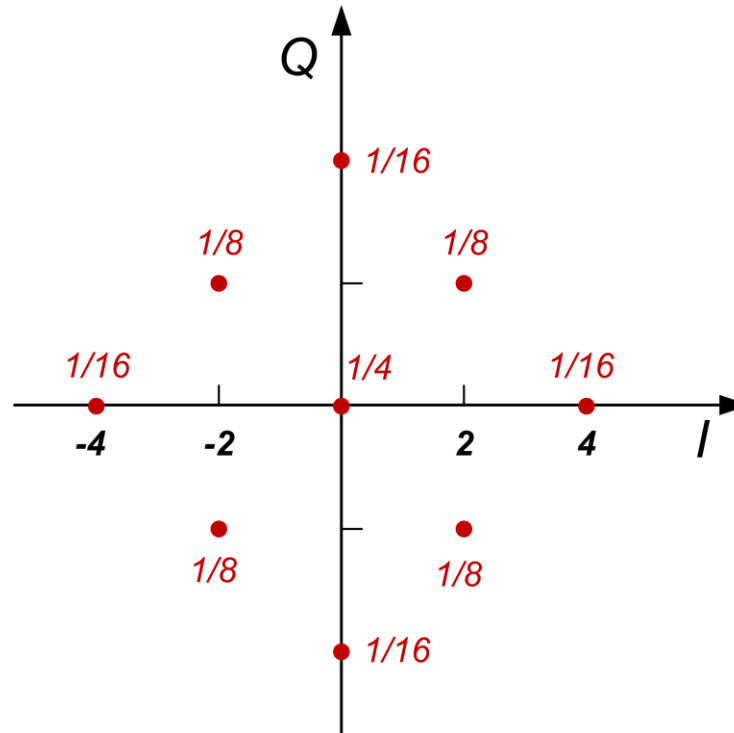
$$\begin{aligned}
 x(t) &= \left[x_{a-I}(t) + jx_{a-Q}(t) \right] e^{-j2\pi 15 f_0 t} + \left[x_{b-I}(t) + jx_{b-Q}(t) \right] e^{+j2\pi 15 f_0 t} \\
 &= \left\{ \left[x_{a-I}(t) + x_{b-I}(t) \right] \cos(2\pi 15 f_0 t) - \left[x_{b-Q}(t) - x_{a-Q}(t) \right] \sin(2\pi 15 f_0 t) \right\} \\
 &+ j \left\{ \left[x_{a-Q}(t) + x_{b-Q}(t) \right] \cos(2\pi 15 f_0 t) + \left[x_{b-I}(t) - x_{a-I}(t) \right] \sin(2\pi 15 f_0 t) \right\}
 \end{aligned}$$

Then, we create an AltBOC *binary* signal by using binary subcarriers

$$\begin{aligned}
 x_{AltBOC}(t) &= \left\{ \left[x_{a-I}(t) + x_{b-I}(t) \right] \text{sgn} \left[\cos(2\pi 15 f_0 t) \right] - \left[x_{b-Q}(t) - x_{a-Q}(t) \right] \text{sgn} \left[\sin(2\pi 15 f_0 t) \right] \right\} \\
 &+ j \left\{ \left[x_{a-Q}(t) + x_{b-Q}(t) \right] \text{sgn} \left[\cos(2\pi 15 f_0 t) \right] + \left[x_{b-I}(t) - x_{a-I}(t) \right] \text{sgn} \left[\sin(2\pi 15 f_0 t) \right] \right\}
 \end{aligned}$$

BUT the signal is not constant-amplitude, yet

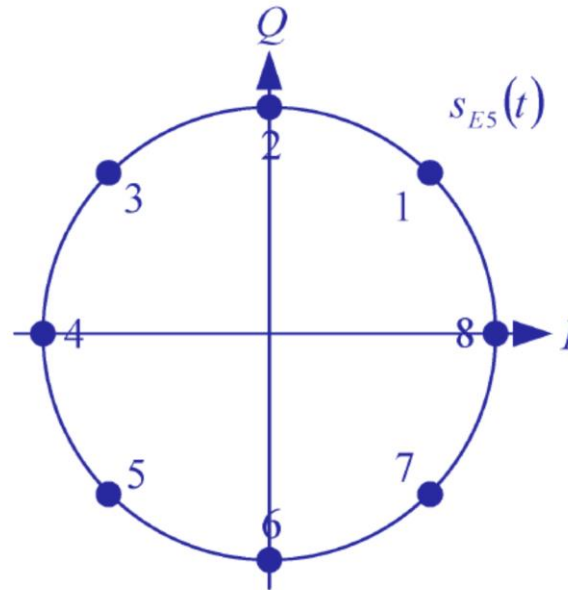
AltBOC amplitude



The labeling represents the relative frequency of each complex value. The average constellation power is 16.

Constant-Amplitude AltBOC feature

$$s_{E5}(t) = \exp\left(j \frac{\pi}{4} k(t)\right) \quad \text{with} \quad k(t) \in \{1, 2, 3, 4, 5, 6, 7, 8\},$$



**Modified AltBOC (MBOC) (constant) Amplitude:
looks like 8PSK**

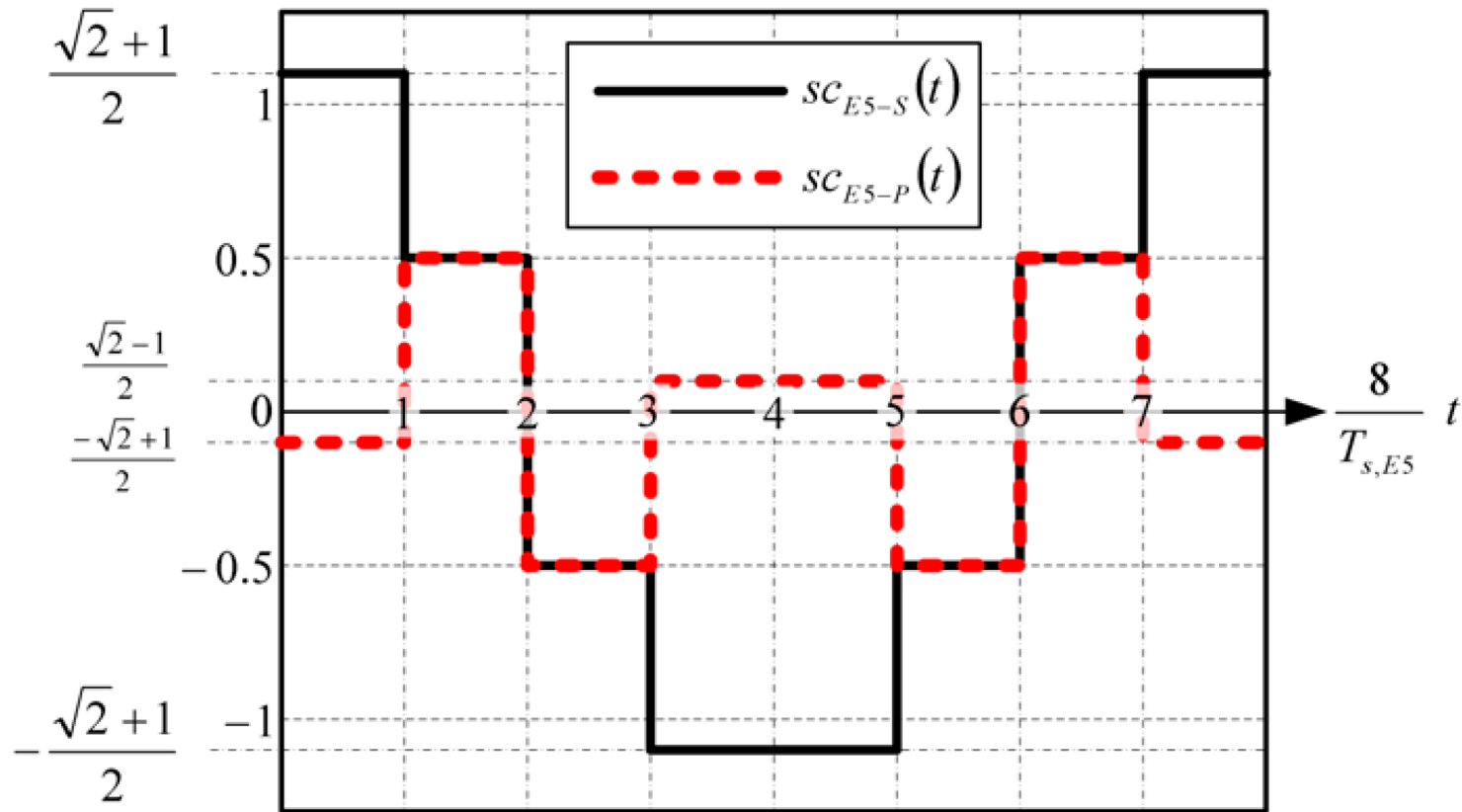
Intermodulation signal

Need to add an *intermodulation* component to keep amplitude constant

$$\begin{aligned}
 x_{MBOC}(t) = & \left\{ [x_{a-I}(t) + x_{b-I}(t)] \operatorname{sgn}[\cos(2\pi 15 f_0 t)] - [x_{b-Q}(t) - x_{a-Q}(t)] \operatorname{sgn}[\sin(2\pi 15 f_0 t)] \right\} \\
 & + j \left\{ [x_{a-Q}(t) + x_{b-Q}(t)] \operatorname{sgn}[\cos(2\pi 15 f_0 t)] + [x_{b-I}(t) - x_{a-I}(t)] \operatorname{sgn}[\sin(2\pi 15 f_0 t)] \right\} \\
 & + I(t; x_{a-I}, x_{b-I}, x_{a-Q}, x_{b-Q})
 \end{aligned}$$

The expression of I is too complicated to be computed real-time; the signal is generated through a **Look-Up Table (LUT) approach** (ROM)

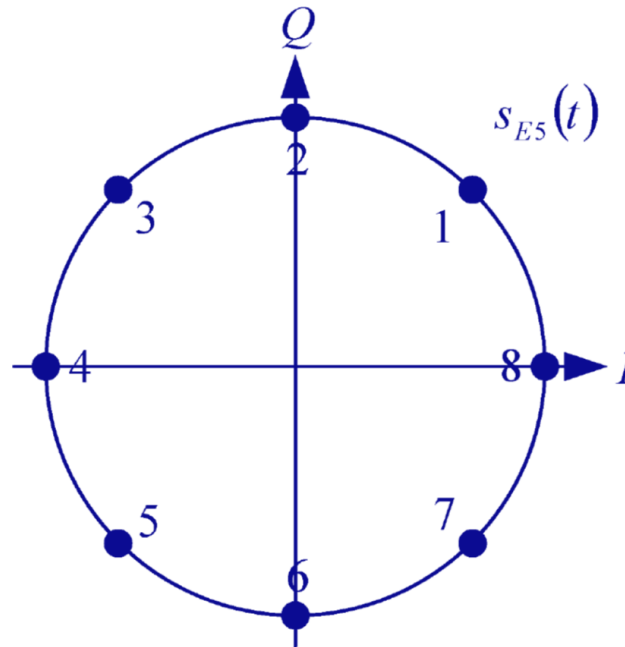
AltBOC Waveforms



The $15f_0$ subcarrier period $T_{s,E5}$ is split in 8 time-slots on which the signal is constant

I/Q Diagram of AltBOC

$$s_{E5}(t) = \exp\left(j \frac{\pi}{4} k(t)\right) \quad \text{with} \quad k(t) \in \{1,2,3,4,5,6,7,8\},$$



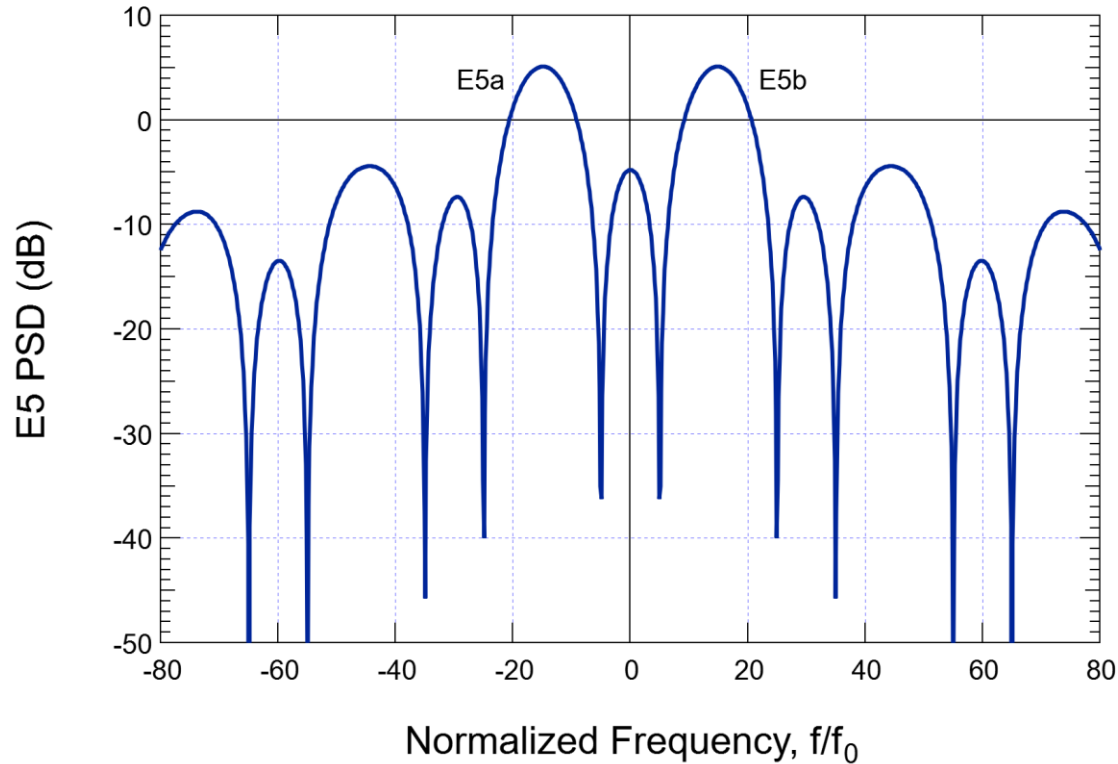
The phase index k changes with time slot-by-slot (every $T_{s,E5}/8$) according to the combination of the binary values of the 4 channels in the same time period. The 4 values are the address of the LUT, k is the contents

Galileo E5 – AltBOC LUT

		Input Quadruples															
eE5a-I		-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
eE5b-I		-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1
eE5a-Q		-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
eE5b-Q		-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
$t' = t \text{ modulo } T_{s,E5}$		k according to $s_{E5}(t) = \exp(jk\pi/4)$															
i_{T_s}	t'																
0	$[0, T_{s,E5}/8[$	5	4	4	3	6	3	1	2	6	5	7	2	7	8	8	1
1	$[T_{s,E5}/8, 2 T_{s,E5}/8[$	5	4	8	3	2	3	1	2	6	5	7	6	7	4	8	1
2	$[2 T_{s,E5}/8, 3 T_{s,E5}/8[$	1	4	8	7	2	3	1	2	6	5	7	6	3	4	8	5
3	$[3 T_{s,E5}/8, 4 T_{s,E5}/8[$	1	8	8	7	2	3	1	6	2	5	7	6	3	4	4	5
4	$[4 T_{s,E5}/8, 5 T_{s,E5}/8[$	1	8	8	7	2	7	5	6	2	1	3	6	3	4	4	5
5	$[5 T_{s,E5}/8, 6 T_{s,E5}/8[$	1	8	4	7	6	7	5	6	2	1	3	2	3	8	4	5
6	$[6 T_{s,E5}/8, 7 T_{s,E5}/8[$	5	8	4	3	6	7	5	6	2	1	3	2	7	8	4	1
7	$[7 T_{s,E5}/8, T_{s,E5}[$	5	4	4	3	6	7	5	2	6	1	3	2	7	8	8	1

Input values of the 4 components in a chip time

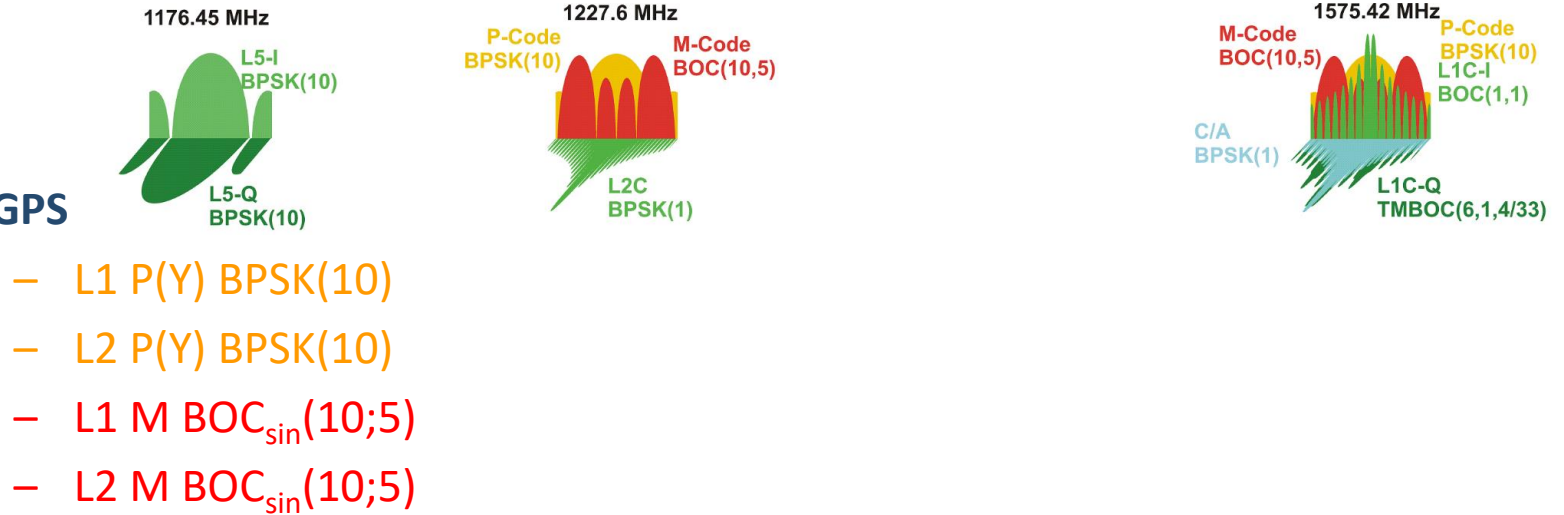
Temporal sequence of the 8 phase values to be generated in the same chip time



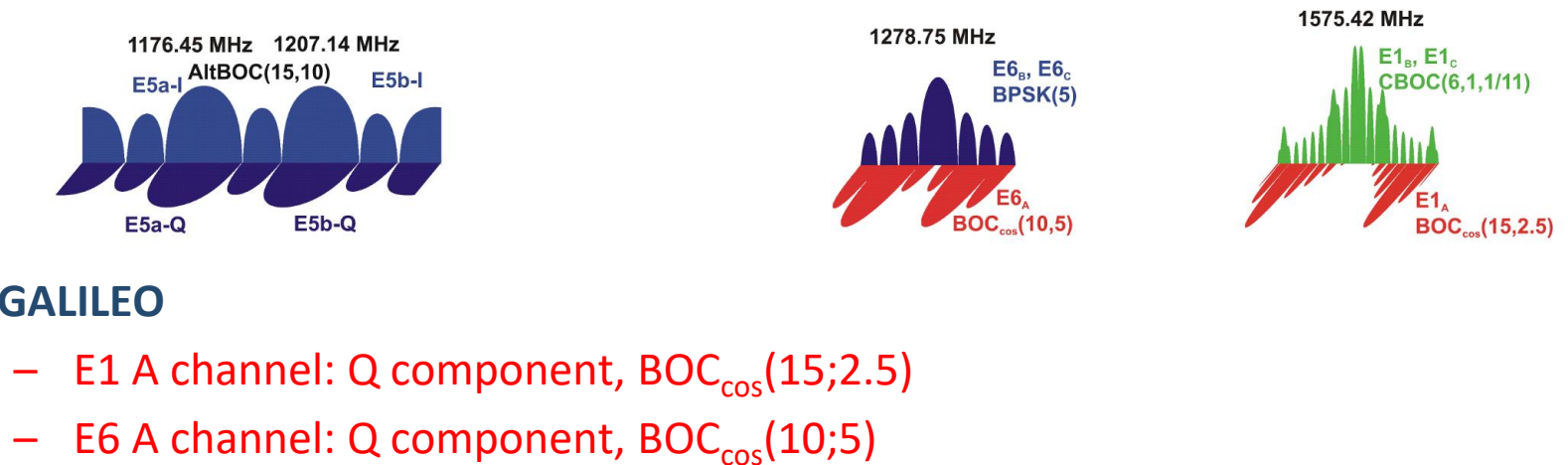
$$S_{E5}(f) = \frac{4 / (10f_0)}{(\pi f / 10f_0)^2} \frac{\cos^2(\pi f / 10f_0)}{\cos^2\left(\frac{\pi f}{30f_0}\right)} \left[\cos^2\left(\frac{\pi f}{30f_0}\right) - \cos\left(\frac{\pi f}{30f_0}\right) + 2 - 2\cos\left(\frac{\pi f}{30f_0}\right)\cos\left(\frac{\pi f}{60f_0}\right) \right]$$

Governative (Encrypted) PRS Channels

- **GPS**



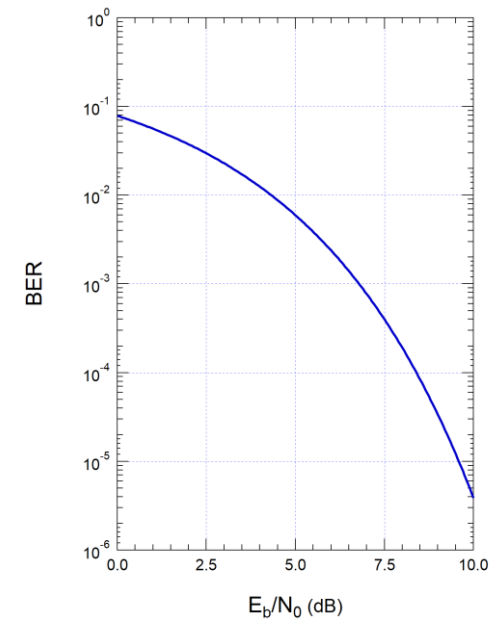
- **GALILEO**



Navigation Data Reception

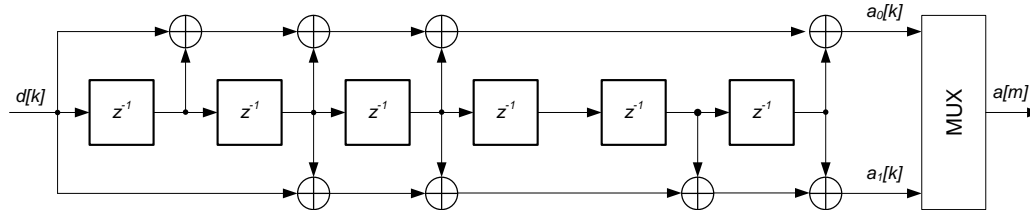
- **Sample BER computation (communications link budget) for GPS L1 C/A**

- Satellite RF power: $P_T = 25.6 \text{ W} = 14 \text{ dBW}$
- TX Antenna gain (max): $G_T = 12 \text{ dB (dBi)}$
- EIRP (max): $\text{EIRP} = P_T \text{ (dB)} + G_T \text{ (dB)} = 26 \text{ dBW}$ (about 500 W equivalent)
- Satellite altitude: $r = 20,200 \text{ km}$
- Free-Space Loss: $L = (4\pi r)^2 / \lambda^2 = (4\pi r f_c)^2 / c^2 = 182 \text{ dB}$
- Received Power on Earth (nominal): $C = \text{EIRP} - L = -156 \text{ dBW}$ (no atmospheric attenuation)
- Overall System Noise Temperature: $T = 500 \text{ K}$ (including antenna noise & LNA noise figure)
- Resulting Thermal Noise level: $N_0 = kT = -201.6 \text{ dBW/Hz}$
- RX Antenna Gain (handheld): $G_R = -1 \text{ dB (dBi)}$
- Receiver C/N_0 ratio: $C/N_0 = C + G_R - N_0 = 45 \text{ dB-Hz}$.
- E_c/N_0 ratio: $T_c C/N_0 = -15 \text{ dB}$ ($T_c = (1/1.023) \mu\text{s}$)
- E_b/N_0 ratio: $T_b C/N_0 = 28 \text{ dB}$ ($T_b = (1/50) \text{ s}$) (**Very GOOD**)
- BER with matched filter: $Q\left(\sqrt{2E_b / N_0}\right) \approx 0$



BER curves

Huge gaps because of
 i) different bit-rate,
 ii) CODING GAIN



BER

